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IMPROVING THE ACCURACY OF CENTRAL DIFFERENCE SCHEMES

Eli Turkel

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IMPROVING THE ACCURACY OF CENTRAL DIFFERENCE SCHEMES

Eli Turkel[1]
School of Mathematical Sciences
Sackler Faculty of Exact Sciences
Tel-Aviv University

ABSTRACT

Central difference approximations to the fluid dynamic equations require an artificial viscosity in order to converge to a steady state. This artificial viscosity serves two purposes. One is to suppress high frequency noise which is not damped by the central differences. The second purpose is to introduce an entropy-like condition so that shocks can be captured. These viscosities need a coefficient to measure the amount of viscosity to be added. In the standard scheme, a scalar coefficient is used based on the spectral radius of the Jacobian of the convective flux. However, this can add too much viscosity to the slower waves. Hence, we suggest using a matrix viscosity. This gives an appropriate viscosity for each wave component. With this matrix valued coefficient, the central difference scheme becomes closer to upwind biased methods.

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I. Introduction

In recent years, central difference schemes have been used with much success to solve transonic flow problems about aerodynamic shapes. These schemes are second order accurate for sufficiently smooth meshes and have an added artificial viscosity to stabilize the scheme and reach a steady state. This artificial viscosity is usually a blend of two terms. One is a fourth difference that stabilizes the even-odd modes that appear with central differences and constants coefficients. Without this viscosity, one cannot reduce the residual beyond some level because of a remaining high frequency mode. The second viscosity term is a nonlinear second difference that limits oscillations in the neighborhood of shocks. A nonlinear shock detector preserves the second order accuracy of the scheme in smooth regions.

An advantage of the artificial viscosity approach is that it allows the user control over the amount of dissipation. Nevertheless, one sometimes finds that there is too much dissipation in the numerical solution. Changing the global constants that appear in the formulas is not sufficient to construct an artificial viscosity that is appropriate in both the shocked and smooth regions of the flow. For some problems, we need to severely limit the viscosity in some smooth regions, e.g., near the trailing edge, while still maintaining stability near the shocks. Hence, although the standard artificial viscosity works well in most cases, it is not sufficiently flexible to handle more delicate problems.

In order to improve the existing artificial viscosity, we shall make use of ideas used in the construction of upwind schemes. In particular, we shall replace the scalar coefficient in the artificial viscosity by a matrix. To prevent difficulties near stagnation points, a cutoff is introduced that depends on the spectral radius of the matrix. By varying the cutoff, one can obtain an appropriate average between the original scalar viscosity and the new matrix viscosity. Since the matrix viscosity reduces the amount of smoothing on the slower waves, it will improve the total accuracy of the scheme.

II. Finite Volume Formulation and Artificial Viscosity

The Euler equations for an inviscid compressible flow can be written in divergence form as

$$\frac{\partial Q}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} = 0 \tag{1}$$

where

$$Q = (\rho, \rho u, \rho v, \rho w, E)^t \tag{2a}$$

$$f = (\rho u, \rho u^2 + p, \rho uv, \rho uw, (E+p)u)^t \tag{2b}$$

$$g = (\rho v, \rho uv, \rho v^2 + p, \rho vw, (E+p)v)^t \tag{2c}$$

$$h = (\rho w, \rho uw, \rho vw, \rho w^2 + p, (E+p)w)^t \tag{2d}$$

and for an ideal gas

$$p = (\gamma - 1)[E - \rho(u^2 + v^2 + w^2)/2]. \tag{2e}$$

We can also write (1) in the form

$$\frac{\partial Q}{\partial t} + div(F) = 0. \tag{1b}$$

We integrate (1) over a three dimensional cell and consider $Q_{i,f,k}$ as an approximation to the average of Q over the cell. Hence,

$$\frac{\partial Q_{i,j,k}}{\partial t} + \frac{\int\int\int divF dV}{\int\int\int dV} = 0$$

or using the divergence theorem,

$$\frac{\partial}{\partial t}(VQ)_{ijk} + \int\int \vec{F} \cdot n dS = 0. \tag{3}$$

Hence, the time change of the average Q is governed by the fluxes entering and leaving the cell.

This finite volume approach leads to a pure central difference method for Cartesian grids. Though this scheme is stable for constant coefficient hyperbolic equations it is subject to instabilities that will prevent the convergence to a steady state. To enhance this convergence a fourth difference viscosity is added to the scheme. The fourth difference causes oscillations in the neighborhood of shocks. Hence, a shock detector is constructed and near the shocks the fourth difference is turned off and only the nonlinear second difference is operative. The total artificial viscosity, $\overline{V}$, is the sum of such second and fourth differences in each coordinate direction.

$$\begin{aligned} \overline{V}_{tot} &= \overline{V}^{\xi}_{i+\frac{1}{2},j,k} - \overline{V}^{\xi}_{i-\frac{1}{2},j,k} + \overline{V}^{\eta}_{i,j+\frac{1}{2},k} - \overline{V}^{\eta}_{i,j-\frac{1}{2},k} \\ &\quad + \overline{V}^{\zeta}_{i,j,k+\frac{1}{2}} - \overline{V}^{\zeta}_{i,j,k-\frac{1}{2}} \end{aligned} \tag{4}$$

Hence it is sufficient to describe these terms in the ξ direction. Since we only take differences at neighboring points the artificial viscosity is always in conservation form.

The first difference is defined as

$$D_{i+\frac{1}{2},j,k} = Q_{i+1,j,k} - Q_{i,j,k} \tag{5a}$$

and the second ξ difference is defined as

$$E_{i,j,k} = D_{i+\frac{1}{2},j,k} - D_{i-\frac{1}{2},j,k}. \tag{5b}$$

We then form the second and fourth differences. In particular the fourth difference is formed as a second difference of a second difference with positive weights [3,8]. Hence,

$$\overline{V}^{\xi}_{i+\frac{1}{2},j,k} = \epsilon^{(2)}_{i+\frac{1}{2},j,k} D_{i+\frac{1}{2},j,k} - (\epsilon^{(4)}_{i+1,j,k} E_{i+1,j,k} - \epsilon^{(4)}_{i,j,k} E_{i,j,k}). \tag{6}$$

Let,

$$\nu_{i,j,k} = \left| \frac{p_{i+1,j,k} - 2p_{i\ j,k} + p_{i-1,j,k}}{p_{i+1,j,k} + 2p_{i\ j,k} + p_{i-1,j,k}} \right|. \tag{7a}$$

$\nu_{i,j,k}$ is used to detect the location of shocks. When $\nu_{i,j,k}$ is large then the fourth difference is reduced. Let,

$$\sigma_{i+\frac{1}{2},j,k} = K^{(2)} \max(\nu_{i-1,j,k}, \nu_{i,j,k}, \nu_{i+1,j,k}, \nu_{i+2,j,k}). \tag{8}$$

We also multiply σ by a function of the Mach number to reduce σ near the surface. Let, $A = \frac{\partial \overline{F}}{\partial Q}, B = \frac{\partial \overline{G}}{\partial Q}, C = \frac{\partial \overline{H}}{\partial Q}$, where $\overline{F}, \overline{G}, \overline{H}$ are the fluxes in the coordinate system (ξ, η, ς). Let λ be a measure of the fluxes. The original code chose λ as

$$\lambda^{\xi} = \lambda^{\eta} = \lambda^{\varsigma} = \rho(A) + \rho(B) + \rho(C) \tag{9a}$$

where ρ is the the spectral radius of the matrix. For problems with a highly stretched mesh it was found [2,3,8] that for increased accuracy one should choose

$$\lambda^{\xi} = \rho(A),\ \lambda^{\eta} = \rho(B),\ \lambda^{\varsigma} = \rho(C). \tag{9b}$$

$K^{(2)}$, $K^{(4)}$ are constants that determine the level of the second and fourth differences. These constants are given as input to the code. Then

$$\epsilon^{(2)}_{i+\frac{1}{2},j,k} = \lambda_{i+\frac{1}{2},j,k} \sigma_{i+\frac{1}{2},j,k} \tag{10a}$$

$$\epsilon^{(4)}_{i,j,k} = \lambda_{i,j,k} \max(0, K^{(4)} - \sigma_{i,j,k}). \tag{10b}$$

In order to imitate the upwind type [7] algorithms we now replace the scalars in (9b) by matrices. Hence,

$$\lambda^{\xi} = |A|,\ \lambda^{\eta} = |B|,\ \lambda^{\varsigma} = |C| \tag{9c}$$

where $|A| = T|\Lambda_{\xi}|T^{-1}$ when $A = T\Lambda_{\xi}T^{-1}$ and Λ_{ξ} is a diagonal matrix with the eigenvalues of A as its entries. This definition of λ can lead to difficulties when an eigenvalue is near zero. Hence, we modified Λ_{ξ} to be

$$\overline{\Lambda}_{\xi} = diag(\max(a_i, q\rho(A)) \qquad a_i = \text{e.v. of } A \tag{11}$$

where q is a specified constant. When $q = 1$, then $\Lambda_\xi = \rho(A) \cdot I$ and so (9b) is recovered. When $q = 0$ then no modification of Λ_ξ is done. In general, we found that $q = 0.2$ gives good results.

We point out that the use of (9c) does not allow for a constant enthalpy solution and so enthalpy damping cannot be used [5].

III. Results

We consider the central difference code with Runge-Kutta time stepping [3,5]. As described above we use a matrix valued artificial viscosity which approximates TVD type schemes [6,7,9]. The fourth difference viscosity is still needed to allow the multigrid acceleration to quickly reach a steady state. We consider inviscid flow about a NACA0012 with $M_\infty = 0.8$, $\alpha = 1.25°$. A 192×32 C mesh is used with 128 points on the airfoil. In [1] it is pointed out that the standard code smears the weak shock on the lower surface. In Figure 1, we plot the result for the standard scheme, but without enthalpy damping. In Figure 2, we show the same case but using the matrix viscosity. The convergence rate is slowed down since the fourth order viscosity is not as strong but the shock on the lower surface is sharper. There is an overshoot on the shock on the upper surface. This is due to fact that the cutoff (10b) is not sufficiently sharp. One way to improve this is to replace (7) by

$$\nu_{i,j,k} = \frac{|p_{i+1,j,k} - p_{i,j,k}| - |p_{i,j,k} - p_{i-1,j,k}|}{|p_{i+1,j,k} - p_{i,j,k}| + |p_{i,j,k} - p_{i-1,j,k}| + \varepsilon} \qquad (7b)$$

so that $\nu_{i,j,k}$ is one at discontinuities. One can also use the matrix coefficient only for the second difference but use a scalar coefficient for the fourth difference viscosity. This accelerates the convergence to a steady state but smears the shocks. The results presented used a four step Runge-Kutta algorithm with the artificial viscosity frozen after the first stage using a matrix viscosity rather than a scalar viscosity adds about 60% to the total CPU time.

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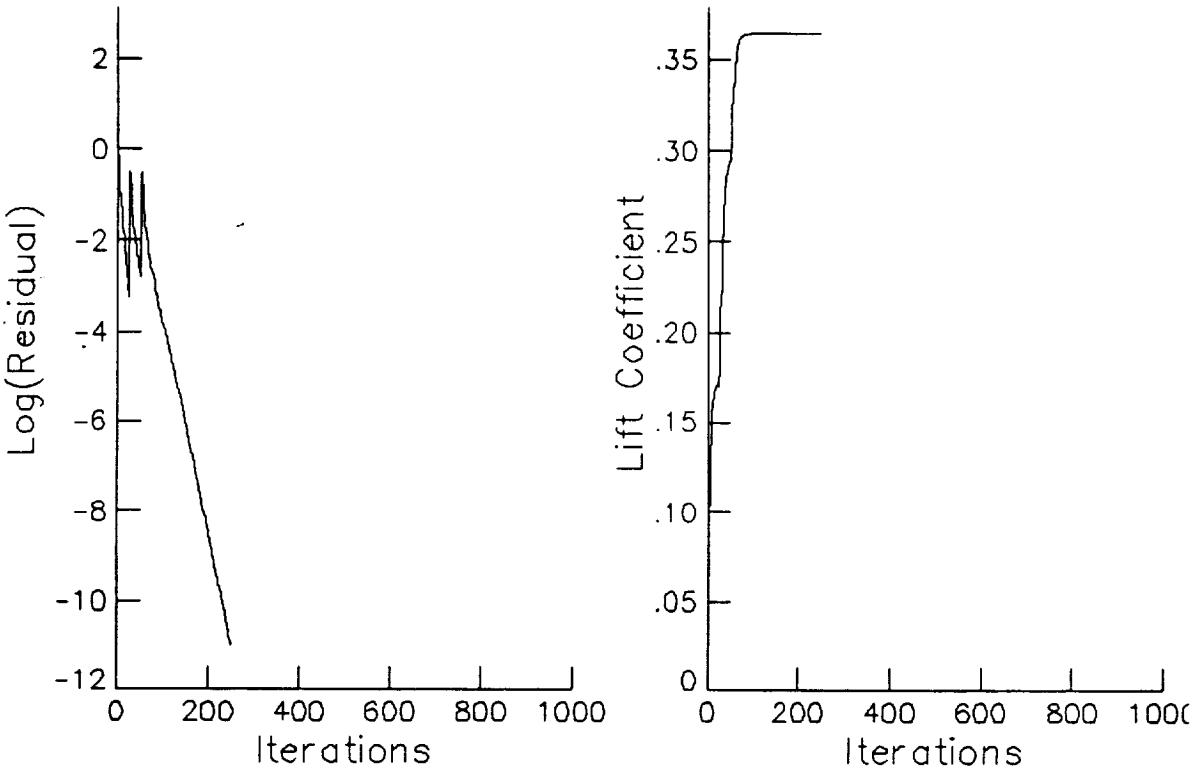


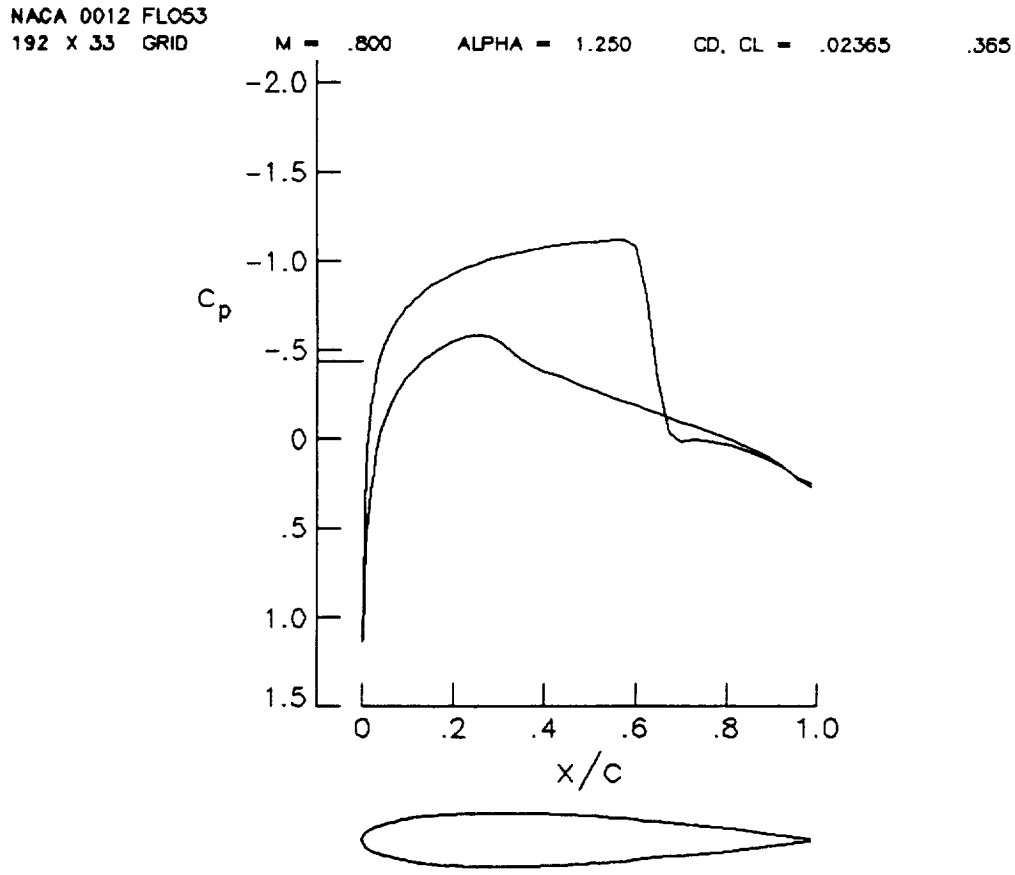
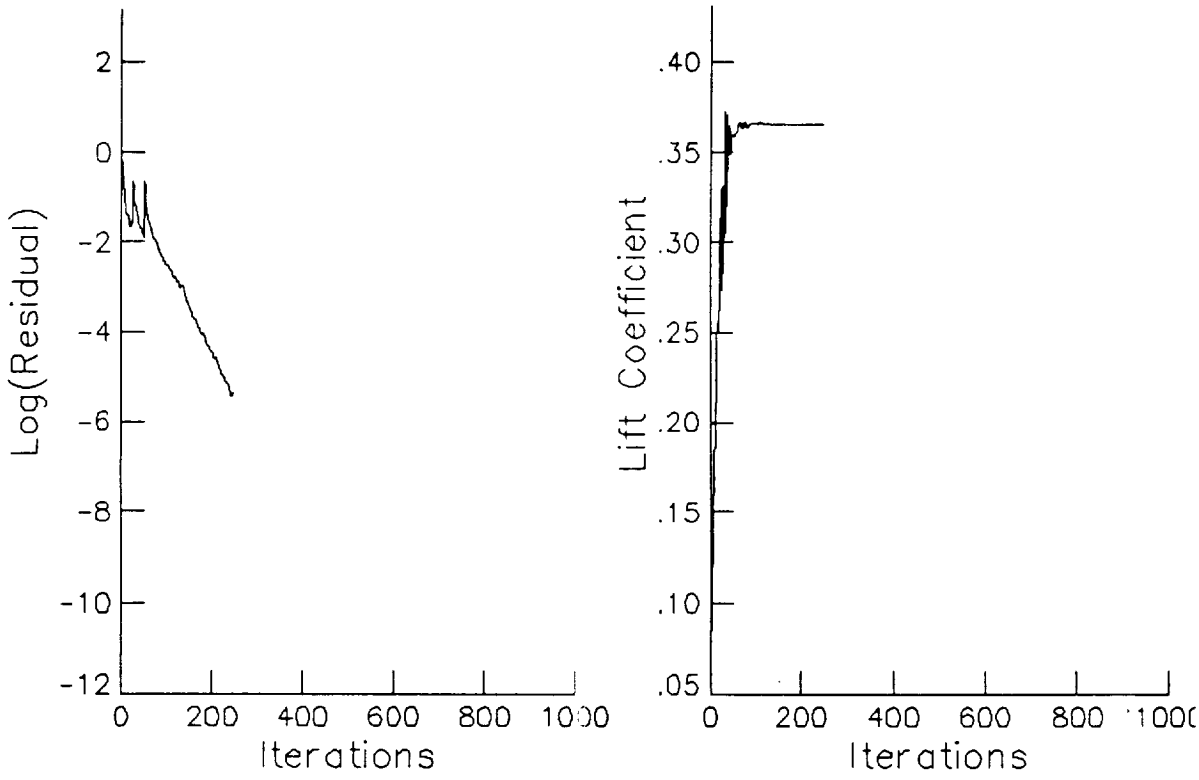


Figure 1: Scalar viscosity



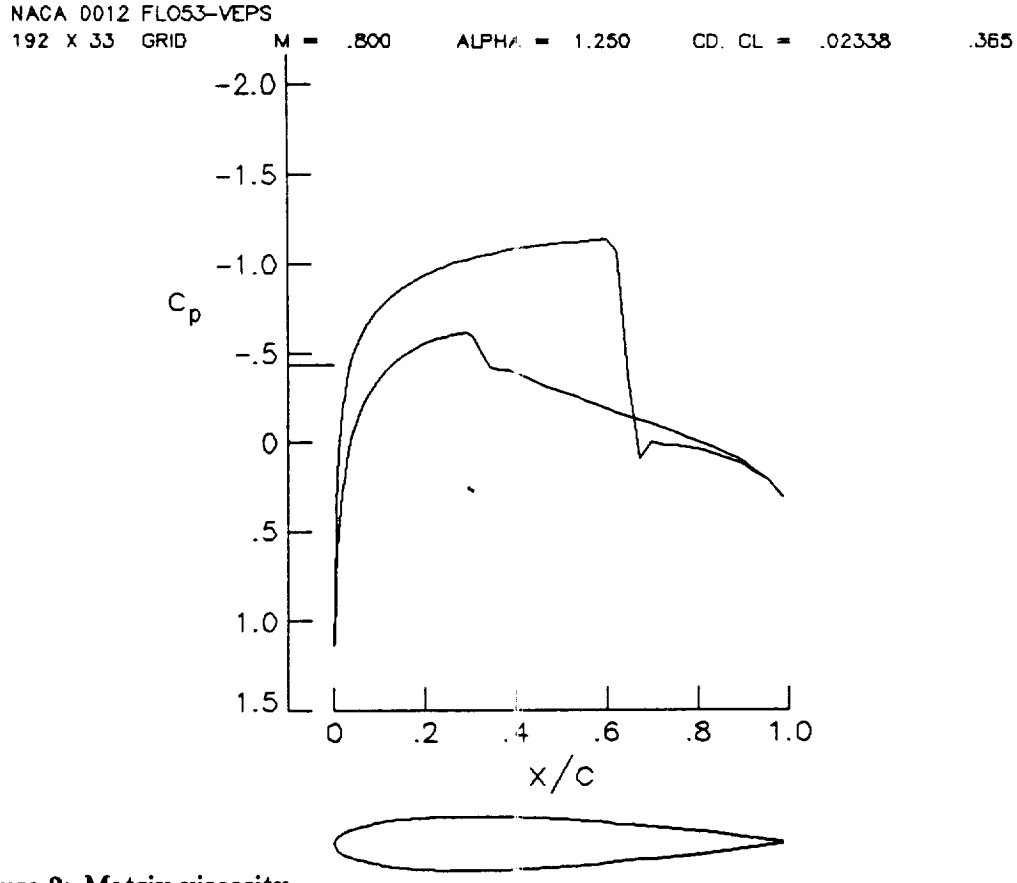


Figure 2: Matrix viscosity

Appendix

We present the matrix viscosity in explicit form for three dimensions in general curvilinear form. Let

$$\xi = \xi(x,y,z), \quad \eta = \eta(x,y,z), \quad \varsigma = \varsigma(x,y,z)$$

be a change of variables from physical space (x,y,z) to a computational space (ξ,η,ς) such that the curvilinear mesh is mapped to a cube. Then (1b) can be rewritten as

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} + \frac{\partial H}{\partial \varsigma} = 0 \tag{A1}$$

where

$$\begin{aligned} F &= \xi_x f + \xi_y g + \xi_z h \\ G &= \eta_x f + \eta_y g + \eta_z h \\ H &= \varsigma_x f + \varsigma_y g + \varsigma_z h. \end{aligned} \tag{A2}$$

We next express (A1) in quasilinear form

$$\frac{\partial Q}{\partial t} + A\frac{\partial Q}{\partial \xi} + B\frac{\partial Q}{\partial \eta} + C\frac{\partial Q}{\partial \varsigma} = 0 \tag{A3}$$

$$A = \frac{\partial F}{\partial Q} \quad B = \frac{\partial G}{\partial Q} \quad C = \frac{\partial H}{\partial Q}.$$

Let

$$\begin{aligned} q &= \xi_x u + \xi_y v + \xi_z w \\ r &= \eta_x u + \eta_y v + \eta_z w \\ s &= \varsigma_x u + \varsigma_y v + \varsigma_z w \end{aligned}$$

be the three contravariant velocities. Also define $h = \frac{E+p}{\rho}$ as the total enthalpy and let $V^2 = \frac{(\gamma-1)}{2}(u^2+v^2+w^2)$ then

$$A = \begin{pmatrix} 0 & a_1 & a_2 & a_3 & 0 \\ a_1V^2 - uq & q-(\gamma-2)a_1u & a_2u-(\gamma-1)a_1v & a_3u-(\gamma-1)a_1w & (\gamma-1)a_1 \\ a_2V^2 - vq & a_1v-(\gamma-1)a_2u & q-(\gamma-2)a_2v & a_3v-(\gamma-1)q_2w & (\gamma-1)a_2 \\ a_3V^2 - wq & a_1w-(\gamma-1)a_3u & a_2w-(\gamma-1)a_3v & q-(\gamma-2)a_3w & (\gamma-1)a_3 \\ -q(h-V^2) & a_1h-(\gamma-1)qh & a_2h-(\gamma-1)qv & a_3h-(\gamma-1)qw & \gamma q \end{pmatrix} \tag{A4}$$

$$a_1 = \xi_x, \quad a_2 = \xi_y, \quad a_3 = \xi_z.$$

For B we get a similar matrix with $a_1 = \eta_x, \quad a_2 = \eta_y, \quad a_3 = \eta_z$ and q replaced by r while for C we have $a_1 = \zeta_x, \quad a_2 = \zeta_y, \quad a_3 = \zeta_z$ and q replaced by s.

Hence, we can find the absolute value of all three matrices in the same way. Let us assume that A has eigenvalues

$$\Lambda = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & 0 & \\ & & \lambda_3 & \\ & 0 & & \lambda_3 \\ & & & & \lambda_3 \end{pmatrix} \tag{A5}$$

where $\lambda_1 = q + \sqrt{a_1^2 + a_2^2 + a_3^2}c, \quad \lambda_2 = q - \sqrt{a_1^2 + a_2^2 + a_3^2}c$ and $\lambda_3 = q$. In practice λ_j, are redefined by (11) to prevent λ_j from approaching zero.

In order to find $|A|$ it is easier to use a two step procedure. Let

$$A_1 = T_1 A T_1^{-1} \tag{A6}$$

be a symmetric matrix. Since we can symmetrize A, B, C simultaneously the same T_1 will work for all three matrices.

$$T_1 = \begin{pmatrix} \frac{V^2}{c} & -(\gamma-1)u/c & -(\gamma-1)v/c & -(\gamma-1)w/c & (\gamma-1)/c \\ -u & 1 & 0 & 0 & 0 \\ -v & 0 & 1 & 0 & 0 \\ -w & 0 & 0 & 1 & 0 \\ -c^2 + V^2 & -(\gamma-1)u & -(\gamma-1)v & -(\gamma-1)w & \gamma-1 \end{pmatrix} \tag{A7}$$

$$T_1^{-1} = \begin{pmatrix} \frac{1}{c} & 0 & 0 & 0 & -\frac{1}{c^2} \\ \frac{u}{c} & 1 & 0 & 0 & -\frac{u}{c^2} \\ \frac{v}{c} & 0 & 1 & 0 & -\frac{v}{c^2} \\ \frac{w}{c} & 0 & 0 & 1 & -\frac{w}{c^2} \\ \frac{h}{c} & u & v & w & \frac{u^2+v^2+w^2}{2c^2} \end{pmatrix}.$$

As expected T_1, T_1^{-1} do not depend on the matrice $a_1, a_2, a-3$.

$$A_1 = \begin{pmatrix} q & a_1c & a_2c & a_3c & 0 \\ a_1c & q & 0 & 0 & 0 \\ a_2c & 0 & q & 0 & 0 \\ a_3c & 0 & 0 & q & 0 \\ 0 & 0 & 0 & 0 & q \end{pmatrix}. \tag{A8}$$

Since A_1 (and B_1, C_1) is symmetric we can diagonalize it with a unitary matrix T_2. T_2 will depend explicitly on a_1, a_2, a_3 and so is different for A_1, B_1, C_1. Let $\mathcal{A} = \sqrt{a_1^2 + a_2^2 + a_3^2} \neq 0$, then

$$T_2 = \frac{1}{\sqrt{2}\mathcal{A}} \begin{pmatrix} \mathcal{A} & a_1 & a_2 & a_3 & 0 \\ -\mathcal{A} & a_1 & a_2 & a_3 & 0 \\ 0 & x_1 & x_2 & x_3 & 0 \\ 0 & y_1 & y_2 & y_3 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2}\mathcal{A} \end{pmatrix} \tag{A9}$$

and

$$T_2^{-1} = T_2^t.$$

The (x_j, y_j) are numbers that satisfy the following equations

$$\begin{gathered} x_1^2 + x_2^2 + x_3^2 = y_1^2 + y_2^2 + y_3^2 = 2\mathcal{A}^2 \\ a_1x_1 + a_2x_2 + a_3x_3 = a_1y_1 + a_2y_2 + a_3y_3 = 0 \\ x_1y_1 + x_2y_2 + x_3y_3 = 0 \\ x_1y_1 + x_2y_2 + x_3y_3 = 0 \\ x_1x_2 + y_1y_2 = -2a_1a_2 \\ x_1x_3 + y_1y_3 = -2a_1a_3 \\ x_2x_3 + y_2y_3 = -2a_2a_3. \end{gathered} \tag{A10}$$

It is difficult to give explicit formula for the x_j, y_j in all cases since some of the a_j may be zero as long as $a_1^2 + a_2^2 + a_3^2 \neq 0$. However, the final formula does not depend on explicitly knowing the a_j. Given T_2 we find that Λ (A5) is given by

$$\Lambda = T_2A_1T_2^{-1} = T_2T_1A(T_2T_1)^{-1}. \tag{A11}$$

We now reverse the process and define

$$|A| = \begin{pmatrix} |\lambda_1| & & & & \\ & |\lambda_2| & & 0 & \\ & & |\lambda_3| & & \\ & 0 & & |\lambda_3| & \\ & & & & |\lambda_3| \end{pmatrix} \tag{A12}$$

where λ_j can be modified eigenvalues of A. Then

$$|A| = (T_2T_1)^{-1}|\Lambda|(T_2T_1). \tag{A13}$$

Let

$$\sigma_1 = \frac{\lambda_1 + \lambda_2}{2} \quad \sigma_2 = \frac{\lambda_1 - \lambda_2}{2} \tag{A14}$$

and define the row vectors

$$\ell_1 = (\gamma - 1)(\frac{u^2 + v^2 + w^2}{2}, -u, -v, -w, 1)$$

$$\ell_2 = (-q, a_1, a_2, a_3, 0). \tag{A15}$$

We then have the matrices

$$I = 5 \times 5 \quad \text{identity member}$$

$$Z_1 = (\ell_1, u\ell_1, v\ell_1, w\ell_1, h\ell_1)^t$$

$$Z_2 = (\ell_2, u\ell_2 + a_1\ell_1, v\ell_2 + a_2\ell_1, w\ell_2 + a_3\ell_1, h\ell_2 + q\ell_1)^t$$

$$Z_3 = (0, a_1\ell_2, a_2\ell_2, a_3\ell_2, q\ell_2)^t$$

and finally,

$$|A| = \lambda_3 I + \left(\frac{\sigma_1 - \lambda_3}{c^2}\right) Z_1 + \frac{\sigma_2}{AC} Z_2 + \left(\frac{\sigma_1 - \lambda_3}{A^2}\right) Z_3 \tag{A17}$$

Because of the simple nature of the matrices Z_j it is easy to multiply $|A|$ times a vector. Define,

$$r_1 = (1, u, v, w, h)^t$$

$$r_2 = (0, a_1, a_2, a_3, q)^t.$$

Let (,) denote the standard inner product, then

$$(\ell_1, r_1) = c^2$$

$$(\ell_1, r_2) = 0$$

$$(\ell_2, r_1) = 0$$

$$(\ell_2, r_2) = \mathcal{A}^2$$

and if x is any column vector, then

$$|A|x = \lambda_3 x + \left[\left(\frac{\sigma_1 - \lambda_3}{c^2}\right)(\ell_1, x) + \frac{\sigma_2}{\mathcal{A}c}(\ell_2, x)\right] r_1 + \left[\frac{\sigma_2}{\mathcal{A}c}(\ell_1, x) + \frac{(\sigma_1 - \lambda_3)}{\mathcal{A}^2}(\ell_2, x)\right] r_2. \quad (A18)$$

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16. Abstract

Central difference approximations to the fluid dynamic equations require an artificial viscosity in order to converge to a steady state. This artificial viscosity serves two purposes. One is to suppress high frequency noise which is not damped by the central differences. The second purpose is to introduce an entropy-like condition so that shocks can be captured. These viscosities need a coefficient to measure the amount of viscosity to be added. In the standard scheme, a scalar coefficient is used based on the spectral radius of the Jacobian of the convective flux. However, this can add too much viscosity to the slower waves. Hence, we suggest using a matrix viscosity. This gives an appropriate viscosity for each wave component. With this matrix valued coefficient, the central difference scheme becomes closer to upwind biased methods.

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